BELLCOMM, INC.
1100 Seventeenth Street, N.W. Washington, D.C. 20036

SUBJECT: Interaction of Space Probes with Planetary Atmospheres: I
Case 233

DATE: June 16, 1967

FROM: R. N. Kostoff

ABSTRACT

This is the first paper of a series which will examine the interaction of different vehicles with the atmospheres of various planets, particularly Mars and Venus. It is assumed that these probes are an integral part of a manned mission to the above planets. Variations in performance which are peculiar to each type of vehicle and which arise from uncertainties in knowledge of the state of the atmosphere will receive particular emphasis. The present paper deals with vehicles which are placed in orbit about the planet. The vehicle parameters of concern are orbit lifetime and drag forces on the vehicle.

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MEMORANDUM FOR FILE

INTRODUCTION

A basic assumption of this paper is that the density of the atmosphere at orbital altitudes may vary continuously with time; in other words, knowledge of the density is subject to an uncertainty $\delta\rho/\rho$, where $\delta\rho$ is the density uncertainty, and ρ is the density. It is also assumed that this uncertainty may be reduced for a manned mission by deploying an impacter probe of the type described in Reference 1 a few hours before periapsis. The effect of a decrease in $\delta\rho/\rho$ in both determination of drag force on, and lifetime of, an orbiting vehicle will now be examined.

ANALYSIS

The atmospheric model chosen for calculations in this paper has the following properties:

A) The atmosphere is isothermal.

B) The density scale height H is constant with change in altitude.

C) Surface pressure (the weight of the atmosphere per unit area) is constant over the surface of the planet.

D) The atmosphere obeys the perfect gas law.

E) The composition of the atmosphere does not vary with time or altitude.

Atmospheric density at any altitude is given by:

$$\rho(Z) = \rho_o e^{-\frac{Z}{H}} \qquad (1)$$

where

$\rho(Z)$ is the density at altitude Z

ρ_o is the density at the planet surface.

ρ_o, in this model, is related to H in the following manner:

Condition C stipulates

$$P_o = \int_0^\infty \rho(Z)g(Z)dZ \tag{2}$$

where

P_o is the constant surface pressure, and $g(Z)$ is the local gravitational potential, assumed equal to g_o (the value at the surface) for this integral. This is justified by the following reasoning.

Consider the two terms under the integral sign:

$$\rho(Z) = \rho_o e^{-\frac{Z}{H}}$$

$$g(Z) = g_o\left(\frac{R_p}{R_p+Z}\right)^2, \text{ where } R_p = \text{planet radius.}$$

In a distance of two scale heights from the surface,

$$\frac{\rho(2H)}{\rho(0)} = e^{-2}$$

$$\frac{g(2H)}{g_o} = \frac{1}{1+\frac{4H}{R_p}} \simeq 1$$

The value of the product of these two terms has decreased to 10% of their value at the surface. This means that the major contribution to the integral has taken place in about two scale heights from the surface. However, g has remained approximately constant. Thus, g may be considered to retain its value at the planet surface, g_o, and may be taken outside the integral sign during the integration.

Insertion of (1) into (2), with subsequent integration, yields:

$$P_o = \rho_o g H \tag{3}$$

or

$$\rho_o = \frac{\text{constant}}{H} = \frac{K}{H} \tag{4}$$

Insertion of (4) into (1) produces:

$$\rho(Z) = \frac{K}{H} e^{-\frac{Z}{H}} \tag{5}$$

A variation of both sides of Equation (5) for constant Z yields:

$$\delta\rho(Z) = \rho(Z) \left|\frac{Z-H}{H^2}\right| \delta H \tag{6}$$

All variational quantities are given the same sign, to finally yield:

$$\frac{\delta\rho(Z)}{\rho(Z)} = \frac{(Z+H)}{H} \quad \frac{\delta H}{H} \tag{7}$$

At the planet surface this becomes:

$$\frac{\delta\rho_o}{\rho_o} = +\frac{\delta H}{H} \tag{8}$$

Application of the perfect gas law at the surface yields:

$$\frac{\delta\rho_o}{\rho_o} = -\frac{\delta T_o}{T_o} = +\frac{\delta H}{H} \tag{9}$$

where T_o is the surface temperature.

Thus, the uncertainty in density in this model is equivalent to an uncertainty in gas temperature. If density uncertainty is derived from scale height uncertainty (Equation 7), the magnifying factor $\left(\frac{Z+H}{H}\right)$ has a strong effect, especially at high altitudes.

Now the relationship of uncertainty in knowledge of density to uncertainty in estimated orbiter lifetime will be presented.

Figure 1 shows the space vehicle orbiting the planet of interest. The symbols are:

P, the planet of interest

R_p, the radius of the planet

R, the distance from the center of the planet to the orbiting vehicle

m, the mass of the orbiting vehicle

g, the gravitational acceleration at R

F_D, the drag force acting on the vehicle

$\bar{a}_\tau$, the tangential deceleration of the vehicle

Drag force is written as:

$$F_D = C_D \cdot \tfrac{1}{2}\rho(R)AV(R)^2 \tag{10}$$

where

C_D is the drag coefficient,

$\rho(R)$ is the density at R,

A is the frontal area of the orbiter,

V(R) is the velocity at R.

It is now assumed that:

A) The vehicle is a point mass at R

B) The orbit is circular

C) Altitude losses occur at the completion of each orbit in a finite step

D) C_D is constant

E) A remains constant

F) The vehicle is injected into orbit at R_p+Z

The energy dissipated by drag force per orbit is given as:

$$E_{DISS} = \int_0^{2\pi} F_D \cdot R \cdot d\theta = 2\pi R\ C_D \cdot \tfrac{1}{2}\rho(R)\ AV^2(R) \tag{11}$$

$\rho(R)$ may be written as:

$$\rho(R) = \rho_o e^{-\frac{R-R_p}{H}} = \frac{K}{H}\ e^{-\frac{R-R_p}{H}} \tag{12}$$

where K is $\frac{P_o}{g_o}$.

Force balance in the radial direction yields:

$$m\frac{V^2}{R} = mg = mg_o \left(\frac{R_p}{R}\right)^2 \tag{13}$$

or

$$V = R_p\sqrt{\frac{g_o}{R}} \tag{14}$$

Substitution of (12) and (14) into (11) yields:

$$E_{DISS} = \pi C_D A g_o R_p^2 \frac{K}{H} e^{-\frac{R-R_p}{H}} \tag{15}$$

The total energy possessed by the orbiting vehicle is the sum of its kinetic and potential energy, i.e.,

$$E_{TOTAL} = \frac{1}{2}mV^2 + mg\ R \tag{16}$$

or

$$E_{TOTAL} = \frac{1}{2}m\ \frac{R_p^2}{R}g_o - mg_o\ \frac{R_p^2}{R} = -\frac{1}{2}\ mg_o\ \frac{R_p^2}{R} \tag{17}$$

The change in energy possessed by an orbiting vehicle with change in altitude is given by:

$$\frac{dE_{TOTAL}}{dR} = +\frac{1}{2}\ mg_o \left(\frac{R_p}{R}\right)^2 \tag{18}$$

Thus, the loss in altitude of the orbiting vehicle after one revolution, due to the dissipative action of the drag force, is the ratio:

$$\Delta R = \frac{E_{DISS}}{\frac{dE}{dR}} = +\frac{2\pi C_D A K R^2 e^{-\frac{R-R_p}{H}}}{Hm} \tag{19}$$

The period of circular orbit is:

$$T = \frac{2\pi R}{V(R)} = \frac{2\pi R^{3/2}}{R_p \sqrt{g_o}} \tag{20}$$

Therefore, the velocity of descent of the orbiting vehicle is given as:

$$V_D = \frac{\Delta R}{T} = +\frac{C_D A}{m} R_p \sqrt{g_o}\, \frac{K}{H}\, e^{-\frac{R-R_p}{H}}\, R^{1/2} \tag{21}$$

The lifetime of the orbiter is approximately:

$$L = \int_{(R_p+Z)}^{R_p} \frac{dR}{V_D} = +\int_{(R_p+Z)}^{R_p} \frac{m}{C_D A}\, \frac{H}{R_p \sqrt{g_o}\, K}\, R^{-1/2}\, e^{\frac{R-R_p}{H}}\, dR \tag{22}$$

$R^{-1/2}$ may be written as:

$$R^{-1/2} = \left[R_p + (R-R_p)\right]^{-1/2} = R_p^{-1/2}\left[1 + \frac{(R-R_p)}{R_p}\right]^{-1/2} \tag{23}$$

If it is assumed that $\frac{R-R_p}{R_p}$ (or $\frac{Z}{R_p}$) is a small quantity (on the order of ten percent or less), then $R^{-1/2}$ may be expanded in a two-term series of the form:

$$R^{-1/2} = R_p^{-1/2} \left(1 - \frac{1}{2} \frac{Z}{R_p}\right) \tag{24}$$

Substitution of (24) into (22) allows a relatively simple closed-form solution to be obtained. Integration of (22) by parts gives:

$$L = +\frac{m}{C_D A} \frac{H}{R_p^{3/2} \sqrt{g_o}\ K} \left(H + \frac{H^2}{2R_p} - He^{\frac{Z}{H}} + \frac{H}{2} \frac{Z}{R_p} e^{\frac{Z}{H}} - \frac{H^2}{2R_p} e^{\frac{Z}{H}} \right) \tag{25}$$

or

$$L = +\frac{m}{C_D A} \frac{H^2}{R_p^{3/2} \sqrt{g_o}\ K} \left(1 + \frac{H}{2R_p} - e^{\frac{Z}{H}} + \frac{1}{2} \frac{Z}{R_p} e^{\frac{Z}{H}} - \frac{H}{2R_p} e^{\frac{Z}{H}} \right) \tag{26}$$

This expression will now be simplified. It is assumed that:

A) $\frac{H}{R_p} <<< 1$

B) $\frac{H}{Z} << 1$

These assumptions reduce (26) to:

$$L = -\frac{m}{C_D A} \frac{H^2}{R_p^{3/2} \sqrt{g_o}\, K} e^{\frac{Z}{H}} \left(\frac{1}{2} \frac{Z}{R_p} - 1 \right) \tag{27}$$

After a variation is performed on both sides of the above equation, with Z held constant, the following expression is obtained:

$$\frac{\delta L}{L} = \frac{2H + Z}{H} \frac{\delta H}{H} \tag{28}$$

Now the variation in lifetime may be related to variation in density by substitution of (7) into (28). This procedure gives:

$$\frac{\delta L}{L} = \frac{2H + Z}{Z + H} \frac{\delta \rho}{\rho} \tag{29}$$

If the density is measured directly as a function of altitude [by, for example, an Aero-Drag probe (Reference 1)], then the uncertainty in orbiter lifetime is proportional to this error in measured density [Equation (29)]. However if, as in the radio occultation experiment (Reference 3), the density at high altitudes is obtained from measurements of density at the surface and scale height as a function of altitude, then the uncertainty in orbiter lifetime is given as the product of scale height uncertainty and the magnifying factor $\frac{Z}{H}$ [Equation (28) for $\frac{Z}{H} >> 1$].

CONCLUSION

Knowledge of the state of the atmosphere a few hours before periapsis of a manned flyby vehicle allows a more accurate determination of the orbiter lifetime for injection at a given altitude. The ΔV expenditure necessary to deploy the atmospheric property measuring probe (described in Reference 1) a few hours before periapsis is calculated in Reference 2. It is here concluded that deployment of the impacter probe, and expenditure of the necessary ΔV to obtain atmospheric information about ten hours before periapsis, is well justified in light of the added information available for improving the planetary orbiter mission.

1014-RNK-caw

R. N. Kostoff

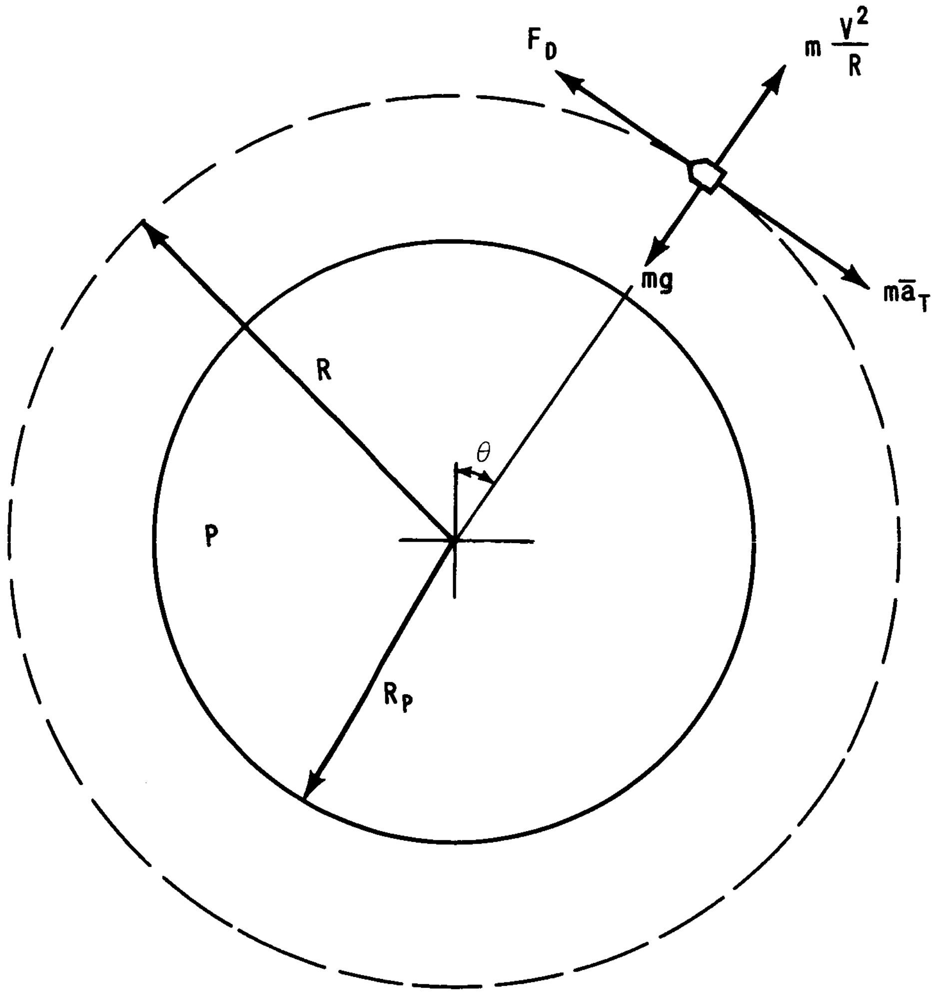
Attachment
Figure 1

REFERENCES

(1) Thompson, W. B., et al, "Experiment Payloads for a Manned Mars Flyby Mission," Appendix B, Bellcomm TR-67-233-1, May 15, 1967.

(2) Kostoff, R. N. and M. Liwshitz, "Analysis of Errors in Targeting an Atmospheric Entry Probe for Impact on Mars," Bellcomm TM-67-1014-3, June 15, 1967.

(3) Kliore, A. and D. Tito, "Radio Occultation Investigations of the Atmosphere of Mars," Journal of Spacecraft and Rockets, Vol. 4, No. 5, pp. 578-583, May 1967.



P	- PLANET
R_P	- PLANET RADIUS
R	- ORBIT RADIUS
θ	- POLAR ANGLE
F_D	- DRAG FORCE
mg	- GRAVITY FORCE
$m\bar{a}_T$	- TANGENTIAL INERTIAL FORCE
$m\frac{V^2}{R}$	- RADIAL INERTIAL FORCE

FIGURE 1 - VEHICLE ORBITING ABOUT PLANET

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